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AUTHOR Ohe, Pia
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ABSTRACT

Twenty children (ages 5 and 6) from each of seven cultural groups (Caucasian, Black, Jewish, Puerto Rican, Chinese, Korean-American and native Korean) were given a copying task of 21 geometric shapes to test the cultural invariancy of Piaget's topological-projective-Euclidean concept acquisition sequence. All subjects were either middle or lower socioeconomic status. Results indicated that the cultural groups did not differ significantly from one another in frequency of topological, projective and Euclidean responses. Subjects attained topological and projective competency to approximately the same degree, but Euclidean competency was still being learned. These findings lend strong support to Piaget's premise concerning the developmental sequence of spatial relations in young children. Neither cultural nor ethnic influences seem to affect the orderly development of definite characteristics and stages in this area of children's cognitive knowledge. (Author/SS)

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HOW DO YOUNG CHILDREN LEARN GEOMETRIC CONCEPTS

A Paper
Presented at the
Ninth Annual International Interdisciplinary
UAP Conference on Piagetian Theory and Its
Implications for the Helping Professions

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by

Pia Ohe

Western Illinois University
Macomb, Illinois 61455

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HOW DO YOUNG CHILDREN LEARN GEOMETRIC CONCEPTS

The devel opment of geometrical concepts among young children begins, according to Piaget (1956, 1960), first, with topological spatial relation (proximity, separation, order, enclosure and continuity), then, secondly, projective space (lines, elementary perspective, "above", "left" and "behind"), and lastly Euclidean space (horizontal and vertical orientation, Perpendicularity, parallalism, "straight", and distance).

Therefore, young children invariably begin with simple, topological spatial understanding long before they come to know the meaning of either projective space or Euclidean space.

The three year old child's drawings are more likely to be "circles" (Guessell, 1925), and furthermore, Di Leo (1973) comments that psychoanalyst may interpret the phenomena as an expression of a child's first perception of his/her mother's breast. However, the children who have never seen a woman's breast either by reason of bottle feeding or institutionalization since birth, likewise begin their graphic activity with "circle".

A "circle", which so easily is percieved and represented by young children does not contain the elements of angles, nor straight lines, therefore is a topological shape. Most psychologists agree that young children often find the "cross" shape much harder to master. The "cross" does not have the topological elements, but it is Euclidean in that the shape contains points (projective), horizontal and vertical line coordinates, and angles (Euclidean).

According to Piaget, in the first year of life, an infant's spatial perception is primarily topological. The most elementary topological

notion is "proximity" i.e., the notion of nearby-ness, the vague awareness that something is near or "out there". The recognition of the primitive functions "separation" and "order" comes next. In the topological sense, "separation" denotes that two proximate elements are not confused or blended but have distinct, separate characteristics.

The relation of "order" comes with series of the mother's movements around the baby (as e.g., in feeding) which the infant perceived as being enclosed by another object or objects, as for instance, the nose and eyes surrounded by the rest of face (Piaget & Inhelder, 1957).

The preoperational stage usually begins around the age of two years and continues until the "concrete operations" stage is established at seven years of age.

Spatial relations at the beginning of this period are called as the "scribbling stage" (Piaget & Inhelder, 1956). The child is able to draw open and closed figures, but diamonds, squares, triangles, circles cannot be distinguished from one another. And rectilinear and curvilinear drawings are not distinguishable at this stage.

After the scribbling stage the four-to-five year old child often draws little men, animals, and houses (Piaget & Inhelder 1956). From these spontaneous drawings one can notice the great difficulty the child encounters with proportions, distances, and angles: a rider remains usupended above his horse, a hat sits in the air above a head, or chimney points toward the house instead of away from it (Piaget & Inhelder, 1956).

This kind of drawing by young children, is merely the child's level of representational space, and not solely a difficulty of motor ability.

For young children, the dimensions of objects change with their positions. The major features in the formation of Euclidean invariance are as follow.

The concepts of parallelism and angles are not percieved by children under five years of age. The child does not understand that parallels stay parallel when rotated. If two equal length rods are placed together, and if one of them is moved or slid slightly, so as to reach past the end of the other, a five year old will probably consider that the "moved" one has changed in length. For estimating distance, the preoperational child will answer that a distance uphill is greater than a (an identical) distance downhill. Apparently it takes more effort to look up or climb up than to come down which seems to indicate that, in this instance, the spatial representation is influenced by motor experience as well as visual. Furthermore the preoperational child would respond that the distance between points

A B

as greater than the distance between points

A B

Therefore, Piaget's preoperational child, ages two to sex, comes to distinguish the topological properties, as a beginning of geometric concepts. Most topological relations become integrated into a stable operational system when a child approaches the age of seven (Piaget, 1970). Projective, Euclidean properties, on the other hand, are said to be learned later, approximately age of nine or ten.

A few Piaget replication studies on the spatial development in terms of specific age trends and the consistency of activities at different stages have been inconclusive. Rivior (1962) reported that the idea of inside/outside

is not developed in most children until the age of six. In her findings, only half the four year olds and two-thirds of the six year olds could identify the contiguous circles. Therefore, she maintained, her findings do not correspond with those of Piaget and Inhelder.

Dodwell (1961) similarly found that the youngest children in his study, frequently drew shapes which were correct in either a projective or Euclidean sense, but not correct in a topological sense. On the other hand, Lovelle, Healey, and Rowland (1962) found broad verification of Piaget's hypothesized topological-projective-Euclidean order, with some exceptions to the sequence as well.

The apparent conflicts in these reports may well be derived from the crude or unrefined nature of the measuring instruments in replication of piaget's experiments. How to refine his clinical experimentation to make it technically valid without destroying his original experiment is a difficult challenge. This study therefore restricts itself to the original twenty-one geometrical shapes of Piaget, as verified to be geometric in the mathematical sense.

The inter-rater-reliability by independent and verifiable scorings, one being the author--the other an established mathematician was .93, thus indicating a consistency of measurement.

MEASUREMENT OF THE SEQUENTIAL CONCEPTUALIZATION

Piaget experimental tasks, the geometrical shapes of copying tasks with their score guide is reproduced in the following figure 1.

Insert Figure 1 about here

For the present experiment, the major hypothesis can be stated as follows: H_0: No significant differences exist across the seven groups in frequency of correct responses to topologica., projective, and Euclidean items. (I.e., all seven groups are at the same stage of development on the topological-projective-Euclidean concept acquisition sequence.)

METHODS

Subjects

The subjects were one hundred forty kindergarten and first grade children, ages five and six. Twenty children were randomly selected from each of seven cultural groups: Caucasian, Black, Jewish, Puerto Rican, Chinese, Korean-American, and Korean-native.

Each of the cultural groups was subdivided into ten boys and ten girls, containing equal age distribution. Since it is almost impossible to locate seven cultural groups in one locality which meet the requirements of equal age distributions, sex, and similar class background, the subjects are of necessity from different localities. The study was therefore conducted at the following locations: Princess Anne Elementary School, Princess Anne, Maryland (Black and Caucasian children); Albert Einstein Academy, Wilmington, Delaware (Jewish); St. Paul Elementary School, Wilmington, Delaware (Puerto Rican); Sacred Heart Convent School, Pusan Korea (Korean-native); the Catholic Sunday School for Koreans, Washington, D.C. (Korean-American); amd Holy Redeemer Elementary School, Philadelphia, Pennsylvania (Chinese).

One set of subjects was limited to one social position as measured by Hollingshead's Two Factor Index (Hollingshead, 1957). All subjects among the Oriental, Black/Puerto Rican, and Caucasian/Jewish group were middle and lower class, except the Jewish children who were mostly middle class.

Procedure

The tests were administered individually during one session of approximately twenty minutes. All tests were given by the author except for the Korean-native. This exception was administered in Pusan, Korea by an established teacher furnished with translations and detailed instructions. For each subject, the face page of the test called for facts on the variables of cultural background, social position, sex, age, formal schooling level, physical and emotional handicaps, and the child's ability to understand and follow the examiner's directions.

Because accuracy was a major concern, the children were allowed ample time making the speed of responses of no relevance. Before beginning each test, the child was given a practice sheet to make certain he/she understood the test directions. Upon demonstrating his/her clear understanding, he/she was permitted to proceed. The attempt has been made to keep varbal instructions to a minimum in order to lessen any ambiguity of instruction or any influence of the examiner.

Result

Table 1 - 7 presents a Chi Square value of the homogeneity of children's responses regarding the three kinds of geometrical items -- topological, projective, and Euclidean levels -- in all seven cultural groups. Only

for the Caucasian group was the difference significant.

TABLE 1

Jewish Group
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi-Square	df	p	.05
top	21	14.33	71.67%		38		
proj	19	15.16	75.79%				
Eucl	16	7.88	39.38%	50.28		N.S.	

TABLE 2

Black Group
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi-Square	df	p	.05
top	21	12.57	62.86%		38		
proj	19	13.16	65.79%				
Eucl	16	8.25	41.25%	25.14		N.S.	

TABLE 3

Puerto Rican Group
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi-Square	df	p	.05
top	21	11.86	59.29%		38		
proj	19	11.37	56.58%				
Eucl	16	7.13	35.63%	27.88		N.S.	

TABLE 4

Chinese Group
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi Square	df	p	.05
top	21	15.67	78.33%		38		
proj	19	16.95	84.74%				
Eucl	16	10.06	50.13%	37.3		N.S.	

TABLE 5

Caucasian Group
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi-Square	df	p	.05
top	21	15.14	75.71%		38		
proj	19	16.21	81.05%				
Eucl	16	6.43	32.19%	89.1			*

TABLE 6

Korean I Group
(Korean-American)
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi-Square	df	p	.05
top	21	15.52	77.62%		38		
proj	19	16.32	81.58%				
Eucl	16	9.69	48.44%	37.44		N.S.	

TABLE 7

Korean-II (Native Korean) Group
(N=20)

Subtest	Possible Score	Mean	Mean Percent	Chi-Square	df	p .05
top	21	14.62	73.10%		38	
proj	19	15.42	77.11%			
Eucl	16	9.13	45.63%	35.38		N.S.

See Abbreviations notes on Table 1.

TABLE 8

Normal Test for Differences of Means
Between Topological Scores
and Projective Scores
(N=140 Subjects)

	Topological (21 items)		Projective (19 items)	
	Sum of x^2	Sum of x	Sum of x^2	Sum of x
	32597	2089	29678	1986
Mean Percent Correct	0.71054422		0.74661654	
Variance	10.582542		10.591124	
Standard Deviation of Difference of Means	0.08708821			

Difference of Means: 0.74661654 - 0.71054422 = .03607232

Critical Ratio: $Z = \frac{.03607232}{.08708821} = 0.4142$ (N.S.)

As Table 8 shows, when topological and projective scores are compared, the differences are not significant at the .05 decision level.

The calculated Chi-square value is 17.79, which is less than the critical value of 21.03 (p=.05). The groups did not differ significantly from one another in relation to frequencies of topological, projective and Euclidean responses.

It is noted, however, that most deviations from expected values take place in the "Euclidean" column. Despite a non-significance finding on a group-by-group tests across the three Piagetian levels, another test was further performed to connect between adjacent levels. Such a test might be useful in verifying whether there is a transition between Piagetian stages in progress.

Therefore, Eculidean means were compared against projective means, and projective means against topological means. Results appear in Tables 8 and 9.

TABLE 9

Normal Test for Differences of Means
Between Projective Scores and
Euclidean Scores
(N=140 Subjects)

	Projective	Euclidean
Sum of x	1986	937
Sum of x^2	29,678	7790
Mean Percent	74.66%	41.83%
Variance	10.5911	10.7512

Standard Deviation of Difference of Means: 0.07388689

Difference of Means: 0.7466142 - 0.4183035 = 0.3283107

Critical Ratio: $Z = \frac{0.3283107}{0.07388689} = 4.443$ ***

*** with 99.99% probability, the difference significant

As Tables 8 and 9 demonstrate, in the hypothesized topological-projective-Euclidean progression, over all percent scores for out subjects--all seven groups, N=140 -- were 71%, 74% and 42% respectively. The first difference, about three percentage points, was not statistically significant; the second, about 30 percentage points, was. The subjects had, by and large, attained topological and projective competency, and to approximately the same degree; but Euclidean competency was still being learned.

Discussion

These findings lend strong seupport to Piaget's premise concerning the developmental sequence of spatial relations in young children. Neither cultural nor ethnic influences seem to affect the orderly development of definite characteristics and stages in this area of children's cognitive knowledge.

When space is considered in terms of the system known in formal mathematics as topology, or so called "rubber-sheet" geometry, the distinciton between a large circle and a small circle is not relevant. Furthermore, a circle is not different from a square, since there is no distinction between straight and curved lines. (Jackson, Robinson, and Dale, 1977)

Because the topological system in an elementary level, deals with such features as continuity, neighboringness, and betweenness, it is rather an easy system for preschoolers, and early elementary school children to percieve and understand. In contrast, the projective system, requires the relationships of distance and directions. Children must learn to separate from their self-centered points of view of space and make the objective estimations and judgements which are a much harder task for children of 6, 7, or even older.

In this study, the perspectives are not tested, as the Piaget's Twenty One Geometrical shapes did not contain this aspect of geometry, but only the "points" and "intersections". In fact, the children, in this study scored the same degree of attainment on topological and projective scores. Therefore, it is regretable to state that the Projective items in this study were not adequate, with the exclusion of perspective geometrical items, and consequently the results were confounding as the Table 8 indicated. It is not unitl well into the elementary school years the measurement of space, such as the coordinate axes and distance, the Euclidean problems are to be learned. And it is clearly evidenced in this study indicating the discrepancy between 74% and 42% former projective scores, and the later Euclidean scores with the significant difference on the 99.99% probability and confident level.

Figure I

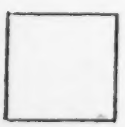
Geometrical Shapes Copying Tasks

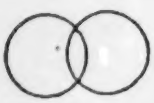
Code: t = topological properties

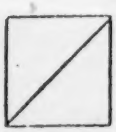
p = projective properties

e = Euclidean (metiric) properties

(3 mm error is allowed.)

	t. Boundary-closed; little circle outside p. Two objects do not touch e. ----------
	t. Boundary-closed; little circle inside p. Two objects do not touch e. ----------
	t. Boundary-closed; little circle on boundary p. Touch in two points e. ----------
	t. Boundary-closed p. ---------- e. Two diameters which divides four equal parts
	t. Boundary-closed p. Four corners touched e. Equal length of four sides

	t. Boundary-closed p. Lines containing three sides touched e. Equal length of the three sides
	t. Boundary-closed p. ---------- e. One side of the diameter is longer
	t. Boundary-closed p. Four corners are touched e. Two length are equal and longer than the other two
	t. Boundary-closed p. Circles don't touch e. Equal size of circles; no ovals
	t. Circles touch; no open loops p. Circles touch in one point e. Equal sizes of circles; no ovals
	t. Circles on Boundary; no open loops p. Circles touch in two points e. Equal sizes of circles; no ovals

	t. Triangle inside (◯ , △ closed figure) p. Triangle, circle don't touch - (△ , three corners touched) e. Equal size for triangle; equal size of circle (no Ovais)
	t. Triangle inside △ , ◯ closed figures p. Triangle touches circle in two points e. △ has one vertex at center of circle, and has two equal sides
	t. Triangle inside; △ , ◯ closed p. Triangle doesn't touch circle; (△ three corners touched) e. Equal size for △ ; this △ is smaller than the above; ◯ not ⬭.
	t. Circle inside triangle; △ , ◯ closed p. Circle touches triangle in three points; △ , three corners touched e. Triangle has three equal sides; ◯ not oval ⬭.
	t. Triangle partly inside, partly outside circle and closed p. Six intersection points, and has three corners touched △ e. Triangle has three equal sides; Circle not oval
	t. Diagonal line completely inside figure; Square, a closed figure p. Four corners touched; two of them have three lines intersecting. (Not ⊟) e. Four equal sides

	t. Line inside figure; closed p. Two corners have three lines intersecting (not or or) e. All five lines equal length
	t. Closed figure p. Four corners touched e. Four equal sides
	t. ---------- p. One intersecting point e. All four arms of cross are same length
	t. ---------- p. One intersection point e. All four arms are same length; orientation changed

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